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Energy-efficient forwarding strategies for Wireless Sensor Networks in fading channels

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Abstract In the context of geographic routing in wireless sensor networks linked by fading communication channels, energy efficient transmission is important to extend the network lifetime. To this end, we propose a novel method to minimize the energy consumed by one bit of information per meter and per second towards the destination in fading channels. Using the outage probability as a measure to maximize the amount of information delivered within a given time interval we decide energy efficient geographic routing between admissible nodes in a wireless sensor network. We present three different approaches, the first is optimal and is obtained by varying both transmission rate and power, the other two are sub-optimal since only one of them is tuned. Simulation examples comparing the energy costs for the different strategies illustrate the theoretical analysis in the cases of log-normal and Nakagami shadow fading. With the method proposed it is possible to obtain a significant energy savings (up to ten times) with respect to fixed transmission rate and power.

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1 Introduction

Applications of Wireless Sensor Networks (WSNs) are numerous in both industry and everyday life. These networks are made up of low-cost sensor devices capable of collecting and transmitting different types of information autonomously. Geographic routing strategies used in wireless communication networks require that each transmitting node is aware of its location, the locations of its neighbors, and the destination. With this information, the message can be routed by choosing intermediate nodes, or relays, which allow the destination to be reached with the maximum possible transmitted information rate and with minimum delay. However, the main design goal of Wireless Sensor Networks is not only to transmit data from a source to a destination, but also to ensure the minimum energy consumption thereby maximizing the lifetime of the network.

In many applications, sensors are deployed in hard-to-reach areas and are not easy to replace. For a sensor, transmission is the most energy-intensive function. Thus, designing energy-efficient routing mechanisms is paramount. This can be achieved by employing energy efficient routing protocols that take into account measurements related to energy efficiency such as Energy per Packet, Average Energy Dissipated, Average Packet Delay, Packet Delivery Ratio, Time until the First Node Dies, and Distance. Many of these protocols are based on the geographic distribution of the nodes and the associated transmission range for each transmitting sensor. However, the idea of a transmission range in fading channels as a bound is an overly simplistic approach. In such cases, the transmission range must be considered as a random variable and must be treated as such to optimize the efficiency of a network.

We assume the problem of optimal joint routing and transmission strategies can be separated, to obtain the same energy savings. There is some MAC layer scheduling mechanism taking care of interference issue; therefore energy consumption is independent among all hop-by-hop transmissions. Then, given two nodes at distance d and a fixed amount of available energy for transmitting information, the case treated in this paper consists on maximizing the amount of information delivered within a given time interval by tuning the transmitter power, the rate of information, or both together when the transmission is affected by fading.

In the presence of fading, the signal to noise ratio is random and the metric used to evaluate the transmission performance depends on the rate of change of the fading. The bit error probability is an appropriate metric that is applied when the fading coherence time is of the order of a symbol time. In this case the signal fading levels are approximately constant and the error correction coding techniques can retrieve the information. However, in the event that the fade signal changes slowly and deeply with respect to the duration of the symbol, they cannot be recovered by encoding. Under these circumstances, a probability of outage is an appropriate metric for assessing transmission performance with independence of the MAC layer is configured.

The channel capacity is a metric for measuring the outage probability, it gives the maximum data rates that can be transmitted over wireless channels with asymptotically small error probability. This metric is a function of the signal-to-noise ratio at the receiver. If it is greater than a given threshold, the transmission can be decoded and vice versa. Thus, in shadow fading channels the capacity is random and the probability that the message cannot be decoded, called outage probability, is an important metric related to energy consumption. The factors to be taken into account in defining the criterion are the outage probability, the expected progress of such information measured in $bits/s/m$, and the expected delay time to deliver the information.

The most accepted path loss model for wireless signal propagation is that where the power attenuation at the receiver is a function of the distance between the transmitter and receiver. Additionally, the signal transmitted through a wireless channel propagation will typically experience random variation due to multipath, giving rise to random variations in the power received at a given distance. Both effects define the fading channel between the transmitter and receiver leading to changes in the signal-to-noise level at the receiver. The simplest way to argue that shadow fading as Lognormal distribution is due to the central limit theorem. However, in a more general context it can be different from Lognormal. We use a Mixture of Gaussians (MoG) distribution as an accurate approximation for several possible envelopes of the signal-to-noise ratio (SNR) distributions of wireless fading channels. In the simulation section, results from the different strategies illustrates the theoretical analysis in the cases of Lognormal and Nakagami shadow fading where a significant saving in energy is obtained when the transmission rate, power, or both simultaneously are tuned.

Using the fact that power consumption is independent among all hop-by- hop transmissions, the optimal and sub optimal designs can be extended to the case of having to select a relay within a subset of admissible relays. Certain geographic routing protocols are able to select, instead of a single relay, a subset of admissible relays as possible next hop relay. For example due to uncertainties or others, there are several possible next hops relays. In this case, the selection strategy among them is found by obtaining the consumed energy for each one in the set. Then, the relay that uses less energy is selected as optimal.

2 Related work

The problem of adapting the transmission of data on wireless channels has been of great interest in recent decades and there has in particular been a remarkable development in the area of sensor networks. In [1] a recent survey analyzes both the energy-efficient and energy-balanced routing protocols for WSNs. In that paper the authors reviewed different energy-efficient and energy-balanced routing protocols that attempt to extend the network lifetime and functionality by minimizing the energy consumption in the network, emphasising the importance of an energy-efficient and energy-balanced routing protocol. In [2] such protocols are classified into four main schemes: Network Structure, Communication Model, Topology Based and Reliable Routing. All these protocols take into account metrics related to energy efficiency such as Energy per Packet, Average Energy Dissipated, Average Packet Delay, Packet Delivery Ratio, Time until the First Node Dies, and Distance. However, the attention in these protocols is focused mainly on strategies that define the best routing of data subject to energy and delay efficiency. By assuming the joint problem of routing and transmission strategies can be separated to obtain the same energy, important energy savings are obtained in [4] by controlling the transmitted power and in [5] by controlling the transmission rate. In [4] the authors propose a transmission power control for energy efficient delivery of information in multihop wireless networks based on finding the optimal signal-to-noise ratio. In [5] energy saving is obtained via lazy schedules that smartly vary packet transmission times. The strategies in [4] and [5] consider gausssian channels. **However, they are not optimal since only the transmission power or rate is tuned, respectively. This approach differs from our work in that we minimize energy by jointly controlling both actions. Moreover, we analyse the case of fading channnels**. The transmission power and rate are designed using the cross-layer approach. For example, the analysis of queuing delay for IEEE 802.16 networks was conducted in [6], [7], and [8] by combining link-layer queuing with physical-layer transmission.

As regards with power and rate control for data transmission in a single transmitter-receiver link several techniques have been proposed. The authors in [9] consider a cross-layer optimization framework for video streaming in multi-node wireless networks in a time-varying interference environment. A joint power control and rate adaptation framework that exploits the time diversity of the wireless channels is designed in such a way that a target delay is achieved. Taking into account the time varying channels and interference, stringent delay constraints, and a certain fairness/satisfaction criterion, they design the link as a stochastic control problem by minimizing a non-linear risk-sensitive cost function. In [10] the authors propose a dynamic rate and power control algorithms for distributed wireless networks that also allow for the congestion levels in a network. Three distributed rate and power control algorithms for wireless networks are presented: an adaptive scheme, a quadratic control scheme, and a robust scheme. The design is achieved by formulating state-space models with and without uncertain dynamics and by determining control signals that help meet certain performance criteria (such as robustness and desired levels of signal-to-interference ratio). **However, the goal in [9] and [10] is to achieve the target delay or a desired signal to noise level. Theses approaches differs from ours is the sense that they do not take into account energy efficiency.**

In a typical mobile radio propagation scenario, the received signal presents power fluctuations due to multipath propagation, known as fading (shadowing). The fading results in a SNR fluctuations around the mean level giving rise to typical distributions like Log-normal, Rayleigh, Nakagami, Weibull and Rice. In most cases an average between some of them results in a complicated pdf that has no closed form expression, as is described in [18]. Thus, a generalized Mixture of Gaussian (MoG) distributions, [19], [20], as an accurate approximation for several possible envelopes, is used.

Fading is important in reliable wireless sensor networks [3]. In [11] experimental results show that many well-designed protocols will fail simply because of fading experienced in a realistic wireless environment. While fading for radio propagation are well understood in the wireless communication community, they are rarely studied in network level research for wireless sensor networks. The authors show that fading can have a significant influence on network performance, notably in multichannel CDMA systems. Similar conclusions are reported in [12]. In [13] it is recognized that fading affects the wireless network and they analyze the connectivity of multihop radio networks in a log-normal shadow fading environment. Assuming the nodes have equal transmission capabilities and are randomly distributed according to a homogeneous Poisson process, they give the minimum node density that is necessary to obtain an almost surely connected network. Due to fading, the amount of information being transmitted between the transmitter and receiver is random, leading to random queuing delays for the buffered packets.

Based on the fact that the consumed energy is a decreasing function of the transmission duration, the total transmission energy can always be reduced by increasing the transmission rate. Associated with a transmission rate, there is a corresponding transmission power that would be required to transmit at a certain rate for a given error probability. Joint optimization of the transmission power, rate, and packet transmission schedules that minimize energy subject to a deadline or a delay constraint have been studied in point-to-point wireless networks by [15],[16], and [17]. The goal is to transmit a set of dynamically arrived packets with the least amount of energy. In all these works is assumed the channel state is known at present time but evolves stochastically in the future as a Markov process. The channel knowledge is based on receiver feedback, pilot measurement or other sophisticated schemes. In [15] and [16] dynamic programming to develop strategies for transmission optimization is used. In [15] the analysis of the buffer occupancy is included. In [17] an optimal-control formulation is used. The main difference between these approaches with our work is that we do not consider channel knowledge in real-time. Instead we assume the distribution of the fading is known avoiding the problem of feedback. On the other hand, the deadline constrained data transmission in our case can be addressed by changing the desired transmission time which is a real-time design variable.

The original contribution of this work is the design of an optimal (energy efficient) information transmission scheme which controls the transmission power and the rate of information in fading channels without knowledge of the channel gain in real-time. The optimization maximizes the information transmitted per unit of time between two WSN nodes separated by a given distance. The energy cost is minimized and realistic fading is taken into account. The optimal transmission design between admissible nodes is used as a metric to decide energy efficient geographic routing. Additional contributions are the possibility to fix the duration for the transmission of the information as well as to derive sub-optimal strategies where only the transmission power or the transmission rate is controlled.

3 Outage Probability

For a complete description of notation used, see Table 1. Let us assume that a source sends r messages per second, and the entropy of a message is H bits per message. The information rate is $R = rH$ bits/second. Shannon's theorem states that i) a given communication system has a maximum rate at which information can be transmitted, known as the channel capacity, C; ii) if the information rate R is less than C, then one can obtain an arbitrarily small error probability by using

Table 1 Notations.

	Name [Units]	Meaning
Channel	C, $[bits/s.]$	Channel capacity
	B, $[KHz]$	Bandwith
	γ	Path loss coeff.
	$\mu_i[dB]$	Mean of ith MoG component relative to $1mW$
	$\sigma_i[dB]$	Stand. dev. of ith component relative to $1mW$
	ω_i	weight of ith MoG component
Power	P_l^{dB}, $[dB]$	Path loss power relative to $1mW$
	P_r^{dB}, $[dB]$	Received relative to $1mW$ to $1mW$
	P_n^{dB}, $[dB]$	Noise power relative to $1mW$
	P^{dB}, $[dB]$	Transmitted power relative to $1mW$
	η^{dB}, $[dB]$	Fading power relative to $1mW$
Power Ratios	SNR^{dB}, $[dB]$	Signal to Noise Ratio at the receiver
	$\overline{SNR}^{dB}$, $[dB]$	Mean of SNR at the receiver
	SNR_R^{dB}, $[dB]$	SNR at rate transmission R
	v, $[dB]$	$\overline{SNR}^{dB} - SNR_R^{dB}$
Transmission	E^{dB}, $[dB]$	Consumed energy relative to $1mWs$.
	T^{dB}, $[dB]$	Time one bits to travel one meter relative to $1s$.
	β, mW	Electronic consumption
	d^{dB}, $[dB]$	Distance between sensors relative to $1m$
	n	Number of retransmissions
	α	Mean number of retransmissions
	$\mathcal{P}_o$	Outage Probability

intelligent coding techniques. In contrast, if R is greater than C due to increased noise, the information can not be recovered. For Gaussian channels, the capacity is given in bits of information per second by

$$C = B\log_2(1 + SNR), \tag{1}$$

where B is the bandwidth of the channel and SNR is the signal-to-noise power ratio at the receiver. Note that the SNR is not in decibels (dB). By using our notations and considering $B = \log(2)$, without loss of generality the capacity is written as,

$$C = \ln(1 + SNR) \tag{2}$$

In order to obtain the expression of SNR, we assume the well-known path loss model affected by fading:

$$Pr^{dB} = P^{dB} - Pl^{dB} + \eta^{dB} + P_n^{dB}, \tag{3}$$

The received power, Pr^{dB}, is equal to the power transmitted, P^{dB}, measured at a reference distance $d_0 = 1m$, minus the path loss, P_l^{dB}, which is function of the distance between nodes, d, plus the random fading process, η^{dB}, plus the thermal noise. The path loss is given by:

$$Pl^{dB} = 10\gamma\log_{10}(d/d_0), \tag{4}$$

where γ is the path loss distance exponent (typically between 2 and 4). All quantities are written in decibels with respect to $1mW$. The fading is represented by a random power signal, η^{dB}, with probability density function that could be Log-normal, Rayleigh, Nakagami, Weibull, Rice, or most likely an average of some of them. P_n^{dB} is the constant power of the thermal additive white Gaussian noise. Therefore, the signal-to-noise ratio in dB at the receiver is given by

$$\begin{aligned} SNR^{dB} &= [P^{dB} - Pl^{dB} + \eta^{dB}]_{signal} - [P_n^{dB}]_{noise} \\ &= \overline{SNR}^{dB} + \eta^{dB}, \end{aligned} \tag{5}$$

where

$$\overline{SNR}^{dB} = P^{dB} - 10\gamma \log_{10}(d/d_0) - P_n^{dB}, \tag{6}$$

Thus, the signal-to-noise ratio is a random variable with mean value $\overline{SNR}^{dB}$. Now, let us assume the transmission rate is $R[bits/s]$. The necessary signal-to-noise ratio level that allows information to be succesfully transmitted at rate R is obtained from the expression of the channel capacity, (2), expressed in dB as follows:

$$SNR_R^{dB} = 10 \log_{10}(e^R - 1) \tag{7}$$

Since the SNR is random, using the concept of channel capacity it is clear that the message is correctly received when $SNR > SNR_R$ and it is lost if $SNR < SNR_R$. Thus, the outage probability is the probability that the SNR is lesser than SNR_R. For example, Figure 1 depicts a possible case where the SNR^{dB} has normal distribution. In this case the outage probability is the area of the distribution in the interval given by $-\infty < SNR < SNR_R$. The outage probability is formally obtained as follows:

$$\begin{aligned} \mathcal{P}_o = \mathcal{P}r\{SNR < SNR_R\} &= \int_{-\infty}^{SNR_R^{dB}} f_\eta(SNR^{dB}) d((SNR^{dB}) \\ &= \mathcal{C}_\eta \left(SNR_R^{dB} \right). \end{aligned} \tag{8}$$

where $\mathcal{P}r$ stands for probability, f_η is the fading probability density function, and $\mathcal{C}_\eta$ is the cumulative distribution function of the fading in dB, evaluated at SNR_R^{dB}.

We use an MoG distribution as an accurate approximation of several possible fading channels. It is formed by a weighted sum of Gaussian functions each with a different mean and variance. The general expression of f_η using MoG is,

$$f_\eta(SNR^{dB}) = \sum_{i=1}^{n_w} \frac{w_i}{\sqrt{2\pi}\,\sigma_i} e^{-\frac{(SNR^{dB} - \mu_i)^2}{2\sigma_i^2}} \tag{9}$$

where w_i are weights of each normal component such that $w_1 + \cdots + w_{n_w} = 1$, μ_i is the mean value and σ_i is the standard deviation of each component.

The MoG distribution can effectively fit any arbitrarily shaped distribution using normal mixture models by fitting the triplets w_i, μ_i, σ_i using the maximum likelihood criteria via the iterative Expectation-Maximization (EM) algorithm, see [19], [20]. Replacing (9) in (8) the outage probability is given by:

$$\begin{aligned} \mathcal{P}_o &= \int_{-\infty}^{SNR_R^{dB}} \sum_{i=1}^{n_w} \frac{w_i}{\sqrt{2\pi}\,\sigma_i} e^{-\frac{(SNR^{dB} - \mu_i)^2}{2\sigma_i^2}} d(SNR^{dB}) \\ &= \sum_{i=1}^{nw} \frac{w_i}{2} \left(erf\left(\frac{\mu - \mu_i - v}{\sqrt{2}\,\sigma_i} \right) + 1 \right), \end{aligned} \tag{10}$$

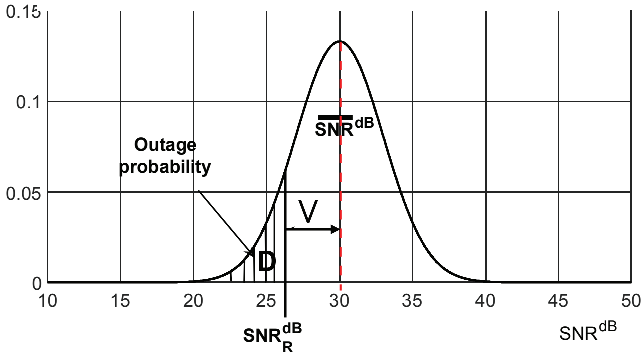

Fig. 1 Probability density function of SNR^{dB} in the case of Gaussian distribution. The area $\mathcal{D} = \{SNR^{dB}|SNR^{dB} < SNR_R^{dB}\}$, is the outage probability.

Notice that $\mathcal{P}_o$ is function of a single variable, v, defined as

$$v = \mu - SNR_R^{dB}. \tag{11}$$

where $\mu = \overline{SNR}^{dB}$ is the mean value of the fading distribution f_η. Note that v is a function of P^{dB} and R through SNR_R^{dB}.

We shall now see how to use this metric to select the relay from a subset of possible candidates in different scenarios. To this end, we need to define the cost function and the decision criterion.

4 Energy consumption

Let us assume that the energy consumed by one bit of information to travel one meter is E in [W. s. / bit. m.]. This energy is proportional to the transmitted power, P, multiplied by the averaged number of re-transmissions for each bit, α, and inversely proportional to the distance, d, and the rate of information transmitted, R. The efficiency of the transceiver is included in P. The constant power supply, β, is added for consumption that is not proportional to the transmitted power itself. The resulting expression is:

$$E = \frac{(P+\beta)\alpha}{dR} \quad \left[\frac{W\ s.}{m.\ bits}\right]. \tag{12}$$

To obtain the value of α let us assume that a number of bits of information must be transmitted from one node to another. In the first attempt there will be a proportion, $\mathcal{P}_o$, that cannot reach the receiving node. Those bits must be re-transmitted, but again a quantity proportional to $\mathcal{P}_o$ will fail and must be re-transmitted and so on until some desired error probability is reached. Thus, the amount of bits transmitted will be greater than the amount of information to be transmitted according to the following calculation:

$$\alpha = 1 + \mathcal{P}_o + \mathcal{P}_o^2 + \cdots + \mathcal{P}_o^{n-1} = \frac{1-\mathcal{P}_o^n}{1-\mathcal{P}_o}, \tag{13}$$

where $\mathcal{P}_o^n$ is the probability that at the end of the re-transmissions, one bit of information cannot reach the next hop. Since $\mathcal{P}_o^n << 1$ the series rapidly converges to $\alpha = 1/(1-\mathcal{P}_o)$ as n increases.

Thus, the energy consumed as follows:

$$E = (P + \beta)T, \tag{14}$$

where T is the time that one bit needs to travel one meter,

$$T = \frac{\alpha}{Rd} \quad \left[\frac{s.}{m.\ bits}\right]. \tag{15}$$

5 Optimal and pseudo-optimal strategies

We assume the two problems of joint routing and information transmission can be separated, to obtain the same energy savings. We assume there is some MAC layer scheduling mechanism taking care of the interference issue. Therefore, power consumption is independent among all hop-by-hop transmissions. Let us first consider the case where some given information needs to be transmitted between two nodes located at distance d from each other. Our strategy, called optimal, consists in finding the pair (P, R) such that the energy consumption to successfully transmit a given amount of information in a given time interval, T, at the given distance, d, is minimal. Since this optimal design is achieved by tuning both R and P it is called PR-control. However, in many cases both variables cannot be tuned and one or both must remain fixed. Hence, there are two possible types of sub-optimal selections. One is by tuning the transmission power while the rate is fixed, called P-control. The other is when the rate is tuned and the power is fixed, called R-control. All strategies will be analyzed.

5.1 Optimal PR-control

The transmitted power can be expressed by replacing Equation (6) in (11) as follows:

$$P^{dB} = SNR_R^{dB} + v + \gamma d^{dB} + P_n^{dB}. \tag{16}$$

The time, T, in decibels with respect to one second, is given by:

$$T^{dB} = \alpha^{dB} - d^{dB} - R^{dB}. \tag{17}$$

Formulated in this way, for each given value of d, T, and P_n there are three unknowns, P, R, and α that need to satisfy two linear equations, (16) and (17) subject to the constraints of minimal energy. Since α is determined by $\mathcal{P}_o$ and this is a function of v, we first obtain v as a minimizer of the energy, E. Then, we replace v in (16) to obtain P and in (10) to obtain $\mathcal{P}_o$. With $\mathcal{P}_o$ we obtain α using

(13). Replacing α in (17) we obtain R. Thus, considering that βT is constant, the minimizer v can be obtained by considering only the energy proportional to he transmitter power in dB,

$$(PT)^{dB} = P^{dB} + \alpha^{dB} - R^{dB} - d^{dB} \tag{18}$$

By replacing (16) in the above and equating to zero the partial derivative with respect to v the following holds:

$$\frac{\partial\,(PT)^{dB}}{\partial v} = 1 + \frac{\partial \alpha^{dB}}{\partial v} = 0 \tag{19}$$

Applying the chain rule, we obtain,

$$1 + \frac{\partial \alpha^{dB}}{\partial \alpha}\frac{\partial \alpha}{\partial \mathcal{P}_o}\frac{\partial \mathcal{P}_o}{\partial v} = 0$$

Taking into account that $\alpha = 1/(1-\mathcal{P}_o)$ and solving the above we obtain the following necessary and sufficient conditions for the minimizer v:

$$\frac{\partial \mathcal{P}_o}{\partial v} = -\frac{\log(10)}{10}(1-\mathcal{P}_o) \tag{20}$$

Remark 1: Note that being a single variable function, the minima are very easy to obtain, either numerically or graphically. Note also that there always exists at least one pair $(v, \mathcal{P}_o(v))$ that satisfies (20). See Appendix 1.

$\diamond$

Using (10) in (20) we obtain the following condition for the minimal energy:

$$\sum_{i=1}^{np} w_i \left(erf\left(\frac{\mu-\mu_i-v}{\sqrt{2}\,\sigma_i}\right) + \frac{\rho}{\sigma_i} e^{\frac{-(\mu-\mu_i-v)^2}{2\sigma_i^2}} \right) = 1 \tag{21}$$

where $\rho = 20/\ln(10)\sqrt{2\pi}$. It should be noted that depending on the fading distribution this equation can have more than one solution. For example, in those cases where the distribution has several relative maximum and minimum, there may be several pairs $(v, \mathcal{P}_o)$ satisfying (21). In those cases is necessary to check each one to choose the optimal. This equation together with (16) and (17) is used to obtain R and P and also the energy consumed by one bit of information to travel one meter, given by (12).

The steps to obtain the optimal values of P, R, and E for single link and, if necessary, the selection from the admissible set can be summarized as follows:

1. First, parameters w_i, μ_i, and σ_i are obtained from the Expectation-Maximization algorithm by fitting the fading distribution. In order to obtain these parameters it is necessary to carry out prior experimental studies. A signal of known power is emitted by the transmitter and measured at the receiver. Power deviations from their average value are recorded and used by the EM algorithm. With these parameters obtain v using (21) and also the outage probability, $\mathcal{P}_o$ from (10).
2. Use the value of $\mathcal{P}_o$ to obtain α from (13) for a given pre-specified desired outage probability.

3. Use α, a given time interval T, and distance d to obtain the optimal value of R from (17).
4. Use v and R together with the path loss coefficient γ, distance d, and noise power P_n to obtain the optimal value of the transmitter power, P, using (16).
5. Finally the energy consumed is obtained by Equation (12).
6. The previous steps allow us to obtain the optimal PR-control for the information transmission between two nodes. The design can be extended in the case of having to select a relay from a subset of admissible relays. Certain geographic routing protocols are able to obtain, instead of a single relay, a subset of admissible relays as possible next hop relay. For example, due to uncertainties, such as location errors or others, there are several possible next hop relays. In this case, the selection strategy among them is found by obtaining the energy consumed for each one in the set by following the steps 1 to 5. Then, the relay that uses the least energy is selected as optimal. Formally, for each admissible next hop relay at distance d_i and fixed time T with consumed energy, E_i, is used in (22) for the optimal selection as follows:

$$\arg\min_{i\in\mathcal{S}} \{E_i\}\,. \tag{22}$$

   This minimization gives the *ith* relay that consumes the least energy to send a bit of information per meter in the time interval T.

5.2 Sub-optimal P-control and R-control schemes

In the case of P-control, the rate R is fixed and given by averaging the values of SNR_R^{dB} obtained from the PR-control for a possible distribution of distances d_i,

$$\bar{R} = \ln(1 + \overline{SNR}_R) \tag{23}$$

where $\overline{SNR}_R = 10^{\mathcal{E}[SNR_R^{dB}]/10}$ and $\mathcal{E}$ means expected value. This parameter can be obtained *a priori* by assuming a possible distribution of distances. When a relay selection is carried out from an admissible subset of possible relays, the sub-optimal strategies select the same relay as in PR-control. The constant v and the outage probability will remain constant for all the strategies.

Similarly, for sub-optimal R-control, the power is fixed equal to the average value of P^{dB} obtained from the PR-control for possible distances d_i.

$$\bar{P} = 10^{\mathcal{E}[P^{dB}]/10}. \tag{24}$$

Note that now, in both sub-optimal designs, the time T is not fixed but varies around the value of the optimal strategy. The energy is obtained with Equation (12) where d_i is given by (22) and P by $\bar{P}$, R by $\bar{R}$ according to the sub-strategy adopted.

Remark 2 It can be stated that always the consumed energy for R-control is closer to PR-control than P-control. This result is demonstrated in the Appendix 2.

◇

Table 2 Simulation parameters.

	Name	Name	Value
Lognormal fading	m	Mean	0 dB
	σ	σ	1.42 dB
Nakagami fading	μ	Shape	0.2
	ω	Spread	0.5
Channel parameters	B	Bandwith	log(2)
	β	Const. power	$[10, 400]\ mW$
	γ	Path loss coeff.	$[2, 4]$
	P_η	Noise Power	$[-20, 0]\ dB$

6 Simulation results

In this section, we present simulations to illustrate the performance and also comparisons between the different strategies and different fading distributions. In the simulations, it is assumed that the admissible relays are pre-selected using a protocol that ensures the advance direction of the message toward the destination. Moreover, the protocol used is able to select relays that are in a narrow band around a straight line to the destination. Any other relay that does not satisfy this condition is discarded, eliminating, in this way, the possibility of considering those that are farther away from the destination than the transmitter, i.e. behind the transmitter. We also assume that each node broadcasts its own location periodically and proactively.

In the example, a Gaussian communication channel affected by shadow fading is simulated. Two different fading distributions are considered, the first with log-normal distribution,

$$f_L(x, \mu, \sigma) = \frac{1}{x\sigma\sqrt{2\pi}} e^{\frac{-(\ln(x)-\mu)^2}{2\sigma^2}} \tag{25}$$

with $\mu = 0dB$ and $\sigma = 1.42dB$. The second is a Nakagami distribution,

$$f_N(x, m, \omega) = 2\left(\frac{m}{\omega}\right)^m \frac{1}{\Gamma(m)} x^{(2m-1)} e^{\frac{-mx^2}{\omega}} \tag{26}$$

where Γ is the Gamma function with shape parameter $m = 0.2$ and scale parameter $\omega = 0.5$.The mean value is $\mu = -9.5dB$. The MoG approximations for Nakagami fading was obtained using the expectation-maximization algorithm with six Gaussian components and a set of 10^5 random samples scaled in dB. The parameter values used for simulations are shown in Table 2. In Figure 2 their scaled histograms and the corresponding MoG approximation are shown for both distributions. The optimal pair $(v, \mathcal{P}_o)$ that satisfies equality (21) is found in Figure 2 for both log-normal and Nakagami fading. In both cases there is only one solution. The value of v is 1.28 for log-normal and -7.77 for Nakagami distribution. Using these values in the MoG representation the outage probability for each distribution is 0.26 for Log-normal and 0.74 for Nakagami.

The simulations were carried out using Matlab. A set of uniformly distributed random distances $d_{k,i}$ in the interval $[1, 150]$ meters were considered, where $k = 1, 2 \ldots, nk$ is the sensor number and $i = 1, \cdots ni$ is the ith stochastic realization.

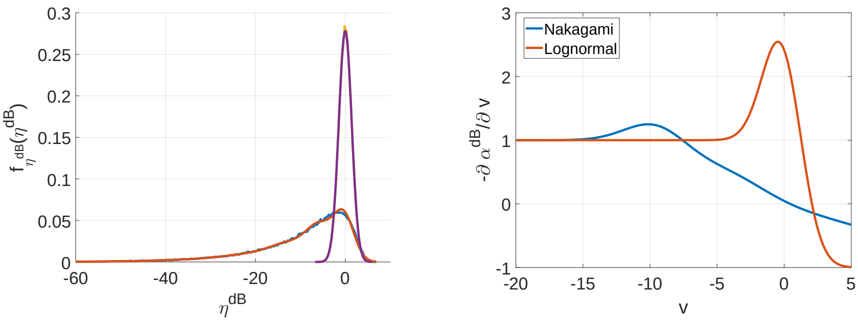

Fig. 2 Right, Lognormal with $\mu = 0dB$ and $\sigma = 1.42dB$ and Nakagami with $m = 0.2$ and $\omega = 0.5$ densities and scaled histograms. Left, pair $(v, \mathcal{P}_o)$ that fullfils Equality (21) or equivalently Equation (19)

The number of $ni = 1000$ repetitions for a set of $nk = 10$ admissible sensors were simulated. For each time T the corresponding elements $R_{k.i}$, $P_{k,i}$, and $E_{k.i}$ of the ni are obtained using Equations (17), (16), and (12), respectively following the steps detailed in subsection 5.1. The channel capacity between the transmitter and receiver has a bandwidth $B = \log(2)\ Hz$. The electronic power is $\beta = 200mW$. The mean values $\bar{R} = \mathcal{E}[R_{k.i}]$ and $\bar{P} = \mathcal{E}[P_{k.i}]$ for sub-optimal strategies are also obtained from the PR-control design. For each of the ni realizations the relay among the $nk's$ with less mean energy is selected for the next hop. Since the design is based on the probability of outage it is not necessary to simulate the coding systems or to compute the errors associated with the decoding, as they are implicit in the outage probability.

The transceiver is often powered by batteries that have a limited amount of energy. Then, we are interested in evaluating the different relay selection strategies that maximize the total information per distance that can be delivered given a fixed amount of energy. Moreover, we are interested in its expected value. Such an amount of rate information in per unit of energy is the inverse of the expected value of the consumed energy for each control strategy in terms of $[m.bits/W.s.]$.

To analyze the efficiency of the proposed strategies, improvements with respect to simultaneous fixed values of both P and R were evaluated. The comparative improvements came from the differences between the performance of the strategies and that obtained with the best possible fixed values of P and R. To this end an exhaustive search of possible simultaneous fixed values of P and R that provide the minimum possible energy, called Fixed Power and Rate, FPR, was carried out *a posteriori*. This means, after the n_i realizations fixed values were found by exhaustive search. Although this method is not possible in a normal operation, it is appropriate for a comparison since it is not possible to find fixed values of P and R that improve the performance achieved with the *a posteriori* adjustment. For this reason the errors that can be obtained in a normal operation are always superior to those obtained in this comparison.

The number of information bits traveling one meter per unit of energy for variations around the nominal values as a function of the period T for both distributions are depicted in the Fig 3 and the relative differences between them depicted in Fig 4.

From the figures the following observations arise:

- Since the PR-control design is based on fixing the delivery time T, it is possible to choose the total time to deliver efficiently all the information.
- The amount of information that can be sent using the full charge of the battery increases with the delay time as expected according to the Shannon capacity theory. In other words, low transfer speeds increase the net amount of information sent for the same energy and this effect coincides with the result in [5]. However, there is a limit to the time which can not grow indefinitely due to energy consumption β which does not depend on R or on P. So, there is a maximum amount of information that can be delivered for any specific time delay. From the different cases analyzed, improvements more than ten times are possible using optimal and suboptimal strategies with respect to fixed values of P and R, as can be seen from Fig 4.
- R-control and P-control have performances very similar to those achieved with PR-control. This is because although one of them is fixed the other is able to compensate to ensure the efficiency of the transmission. However, in Appendix I it is shown that R-control improves P-control.
- The increase in the path loss coefficient, γ, reduces the power in the receiver so that, even with PR control, the information bits traveling one meter per unit of energy decay. The relative difference from the FPR increases as γ increases. This reveals that, under the most extreme conditions, PR-control achieves greater relative advantages. A similar situation occurs with the thermal noise power level, P_n.
- In the case of varying the fixed power consumption, it is seen that the information bits traveling one meter per unit of energy decline with the increase of β as T increases. The relative improvement with respect to the case of FPR increases for smaller β.

6.1 Performance comparison with other approaches

In [4] a transmission power control for energy efficient delivery of information in multihop wireless networks is performed. The authors propose a local power efficiency metric for distributed routing such that at each step, to select the next hop, the transmitter picks the neighbor for which this metric is maximized. Through extensive simulations, they compare the performance of the proposed algorithm with others that are globally optimal: the Dijkstra [21] algorithm which utilizes global knowledge of channel state information to find the shortest end-to-end path in terms of minimal expected power for reliable transmissions; the Joint Distributed Routing and Power Control, JDRPC, proposed by the authors, [4], in which at each step the transmitter picks, as the next hop, the neighbor of with optimal SNR in terms of energy efficiency; the Fixed-transmission-power routing algorithm which allows a transmitting node to choose the next hop in its local neighborhood within a given radius to make the maximal progress toward the sink; and the Distance-based routing, which finds the best end-to-end path with the least expected end-to-end power consumption, utilizing only the distance information between nodes. From exhaustive simulations and comparing the performances of

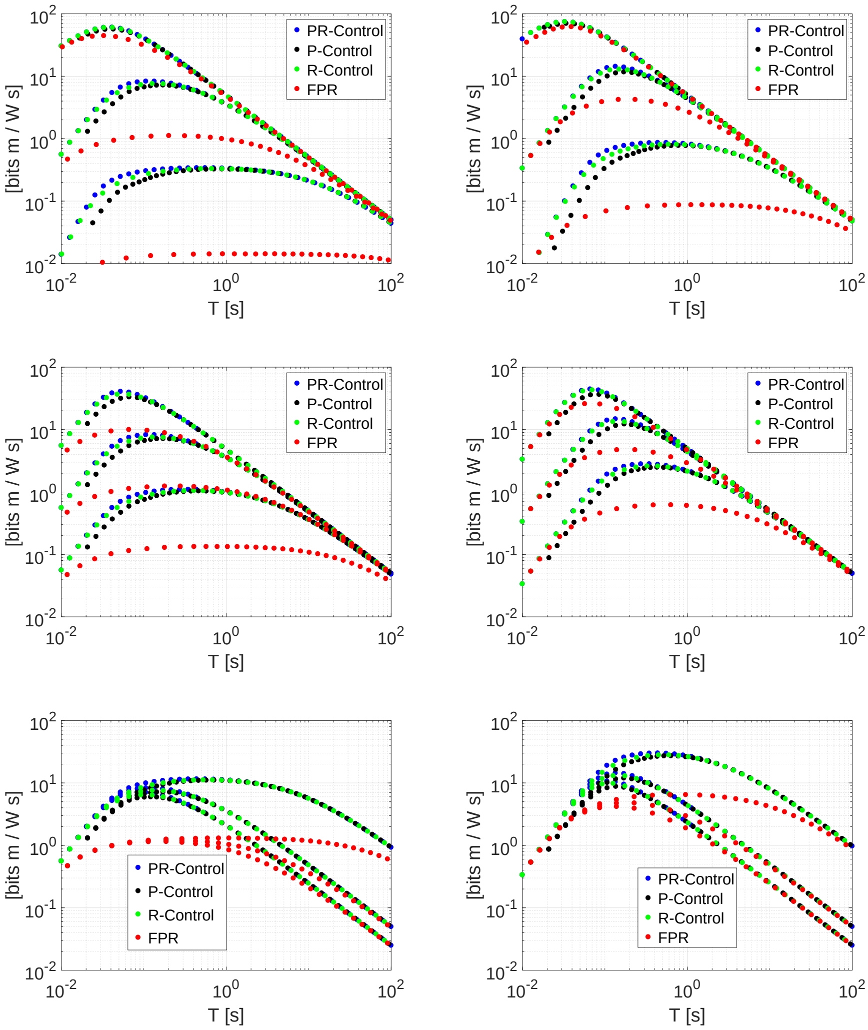

Fig. 3 Number of information bits traveling one meter per unit of energy for variations around the nominal values $\gamma = 3$, $\beta = 200mW$ and $P_n = -10dB$ as a function of the period T. The graphs in the first column (left side) correspond to the log-normal fading and those on the second column (right side) to Nakagami fading. The graphs in the first row show the performance obtained for variations of $\gamma = 2$, $\gamma = 3$ and $\gamma = 4$ (upper, middle and lower curves). The graph of the second row shows the performance obtained for variations of $P_n = 0dB$, $P_n = -10dB$ and $P_n = -20dB$ (upper, middle and lower curves). Finally, the graphs of the third (last) row show the performance for changes in β. At the upper part is depicted the performance for $\beta = 10mW$ $\beta = 200mW$ and $\beta = 400mW$ (upper, middle and lower curves).

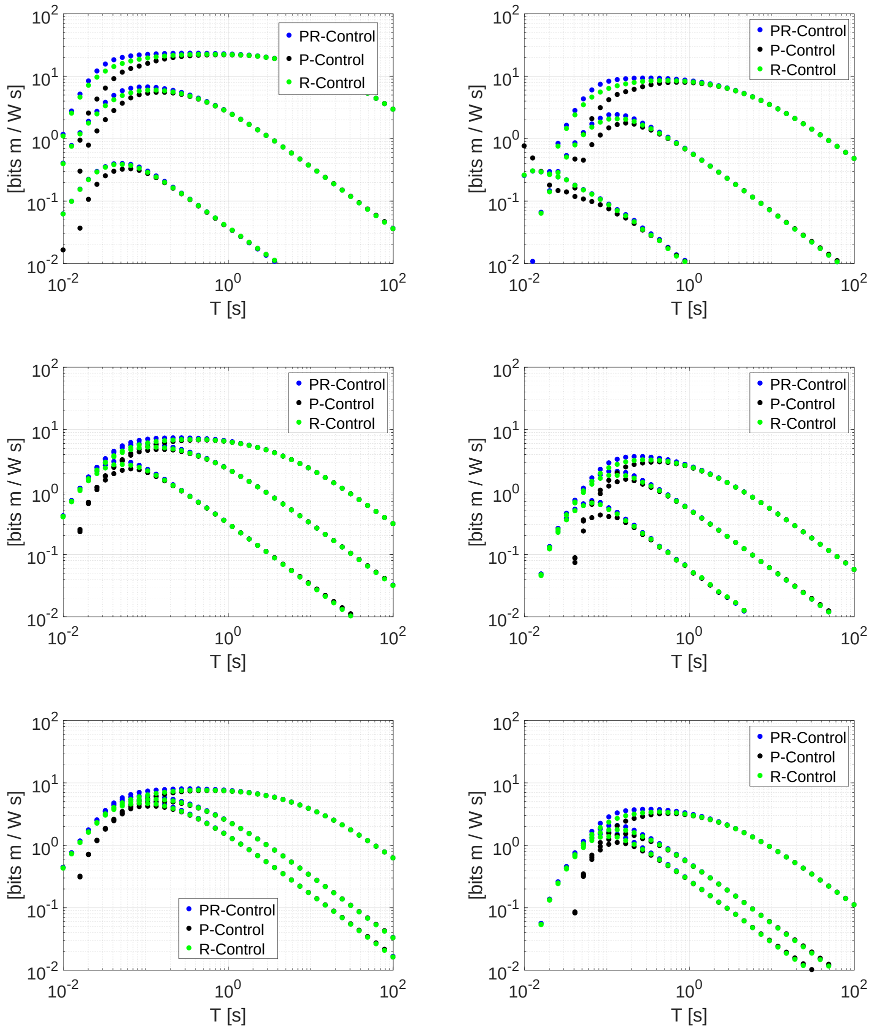

Fig. 4 Relative differences of information bits traveling one meter per unit of energy between strategies and FPR. The graphs on the first column (left side) correspond to the log-normal fading and those on the second column (right side) to Nakagami fading. The graphs in the first row show the relative differences obtained for variations of $\gamma = 4$, $\gamma = 3$, and $\gamma = 2$ (upper, middle and lower curves). The graphs of the second row shows the relative differences obtained for variations of $P_n = -20dB$, $P_n = -10dB$, and $P_n = 0dB$ (upper, middle and lower curves). Finally, the graphs of the third (last) row show the relative differences for changes in β. We set $\beta = 10mW$, $\beta = 200mW$ and $\beta = 400mW$ (upper, middle and lower curves).

the different strategies with respect to the Fixed-transmission-power routing, the authors conclude that JDRPC outperforms the others in up to six times of reduction in power usage. From the simulation analysis of our results it is observed that P-control shows similar performances to those obtained with JDRPC. Moreover, our results show similar improvements are found with respect to other variables such as thermal noise and the path loss coefficient. In the case of low values of the path loss coefficient, improvements are even greater than for power consumption. The performances achieved with JDRPC are even improved by using the R-control and PR-control strategies. An important difference is that PR-control allows one to set the time interval T to carry out the transmission of the information with energy constraints. This does not happen with any of the other strategies. In addition, we show in Appendix I that R-control always exceeds P-control, although the behavior is very similar to optimal PR-control.

7 Conclusions

We have presented three designs for energy efficient transmission in a sensor network under shadow fading channels based on minimal energy consumption. The first is when both the transmission rate and power can be tuned. In the second and third, only the transmission power or the transmission rate is tuned. The criterion is based on computing the outage probability of the information to reach the next hop. The two sub-optimal controlled strategies perform similarly to the optimal, but we have analytically demonstrated that the rate-control is closer to the optimal than the power control. In the optimal case, PR-control, the minimization is performed for a fixed period of time. This implies that for a given amount of information that needs to be sent it is possible to fix the interval delay time with minimum energy. However, in both sub-optimal designs, the time interval time is not fixed but varies very close around the value of the optimal design. In order to evaluate the improvements, simulations were performed using two fading distributions, the Log-normal and the Nakagami and comparing with the case in which the rate and power are fixed. The results shows that even in the case with best possible fixed values of P and R, performances can be improved by up to ten times. **Finally, we leave for future work a robust energy efficient design that does not require knowledge of the probability distribution of fading.**

Appendix I: $\mathcal{P}_o(v)$ is a continuous and monotonically decreasing function of v from $\mathcal{P}_o(-\infty) = 1$ to $\mathcal{P}_o(\infty) = 0$. Then, for any arbitrary value v^* it follows that $\mathcal{P}_o(v^*) \in (0, 1)$, and the derivative in (20) belongs in the interval $(-\log(10)/10, 0)$. Then, invoking the mean value theorem there exists at least one value $\overline{v} > v^*$ and $\underline{v} < v^*$ such that the following holds:

$$\left.\frac{\partial \mathcal{P}_o(v)}{\partial v}\right|_{v=v^*} = -\frac{\log(10)}{10}(1 - \mathcal{P}_o(v^*) \tag{27}$$

$$= \frac{\mathcal{P}_o(\overline{v}) - \mathcal{P}_o(\underline{v})}{\overline{v} - \underline{v}} \tag{28}$$

which proves that there always exists at least one pair $(v, \mathcal{P}_o(v))$ that satisfies (20).

Appendix II: Denoting by $\bar{x}$ the mean value of a random variable x, the subfix o for PR-control, p for P-control, and r for R-control, from (12), considering $\beta = 0$ for simplicity, and taking into account that in all cases the pair $(v, \mathcal{P}_o)$ is the same, which means the comparison is made over the same fading, we obtain the following mean values for the energy in decibels:

$$\bar{E}_r^{dB} = P_r^{dB} - \bar{R}_r^{dB} + \alpha^{dB} - \bar{d}^{dB}, \tag{29}$$

$$\bar{E}_p^{dB} = \bar{P}_p^{dB} - R_p^{dB} + \alpha^{dB} - \bar{d}^{dB}. \tag{30}$$

where R_p and P_r are constant values. From (16) we obtain,

$$P_r^{dB} = \overline{SNR}_{Rr}^{dB} + v + \gamma \bar{d}^{dB} + P_n, \tag{31}$$

$$\bar{P}_p^{dB} = SNR_{Rp}^{dB} + v + \gamma \bar{d}^{dB} + P_n. \tag{32}$$

where $\overline{SNR}_{Rr} = \mathcal{E}[e^{Rr} - 1]$ and $SNR_{Rp} = e^{Rp} - 1$. Replacing in (29) and (30) we obtain,

$$\bar{E}_r^{dB} = \overline{SNR}_{Rr}^{dB} - \bar{R}_r^{dB} + L, \tag{33}$$

$$\bar{E}_p^{dB} = SNR_{Rp}^{dB} - R_p^{dB} + L. \tag{34}$$

where $L = \alpha^{dB} + v + (\gamma - 1)\bar{d}^{dB} + P_n$ is a constant. Since R_r is a design variable that minimize E_r for each value of d, the following property holds: $\overline{SNR}_{Rr} = \mathcal{E}[e^{Rr} - 1] \neq e^{\mathcal{E}[Rr]} - 1$. This optimal can never be satisfied by a constant value R_p for which $\overline{SNR}_{Rp} = \mathcal{E}[e^{Rp} - 1] = e^{\mathcal{E}[Rp]} - 1$. Thus, always fulfils $\bar{E}_r^{dB} \leq \bar{E}_p^{dB}$.

References

1. Ogundile, Olayinka O. and Alfa, Attahiru S., (2017). A Survey on an Energy-Efficient and Energy-Balanced Routing Protocol for Wireless Sensor Networks. Sensors, Vol. 17, 5, pp. 1424-8220.
2. Nikolaos A. Pantazis, Stefanos A. Nikolidakis and Dimitrios D. Vergados, S., (2013). Energy-Efficient Routing Protocols in Wireless Sensor Networks: A Survey IEEE Comm Surveys and Tutorials, V15,2, pp. 551-591.
3. R. Milocco, H. Costantini, S. Boumerdassi, (2014). Improved geographic routing in sensor networks subjected to localization errors. Ad Hoc Networks , V. 13, Part B, 476-486. Elsevier.
4. Chih-ping Li, Wei-jen Hsu, Bhaskar Krishnamachari, Ahmed Helmy, (2005) local metric for geographic routing with power control in wireless networks. Second Annual IEEE Communications Society Conference on Sensor and Ad Hoc Communications and Networks, 26-29 .
5. E. Uysal-Biyikoglu, B. Prabhakar, and A. El Gamal. Energy-Efficient Packet Transmission Over a Wireless Link, (20029. IEEE/ACM Trans. on Networking, 10 , 4, pp. 487-499.
6. R. Iyengar , et al ., Delay analysis of 802.16 based last mile wireless networks , Proc. IEEE Globecom , 2005 , Nov. 2005 .
7. D. Niyato , et al. , Queue-aware uplink bandwidth allocation for polling services in 802.16 broadband wireless networks, Proc. IEEE Globecom , 2005 , Nov. 2005 .
8. D. Rajan , A. Sabharwal and B. Aazhang , Delay-bounded packet scheduling of bursty traffic over wireless channels , IEEE Trans. Information Theory , Vol. 50 , Issue 1, pp. 125-144 , Jan. 2004.
9. A. Ahmad, N. UI Hassan, M.Assaad, and H. Tembine, Joint power control and rate adaptation for video streaming in wireless networks with time-varying interference, IEEE Trans. Vehicular Technology, 65(8): 6315-6329, 2016.
10. A. Subramanian and A. H. Sayed, Joint rate and power control algorithms for wireless networks, IEEE Trans. on Signal Processing, 53(1): 4204-4214, 2005.
11. Liu, B. H., Otis, B. P., Challa, S., Axon, P., Chou, C. T., Jha, S. K. (2008). The impact of fading and shadowing on the network performance of wireless sensor networks. International Journal of Sensor Networks, 3(4), 211.

12. , Lu S. , May, J., and Haines, R. (2015). Effects of Correlated Shadowing Modeling on Performance Evaluation of Wireless Sensor Networks. In Vehicular Technology Conference (VTC Fall), 2015 IEEE 82nd Institute of Electrical and Electronics Engineers (IEEE).
13. Bettstetter, C., Hartmann, C. Connectivity of Wireless Multihop Networks in a Shadow Fading Environment. Wireless Network 11, Springer, 571-579 (2005).
14. Andre Goldsmith, (2005). Wireless Communications. Cambridge University Press.
15. R.A. Berry and R.G. Gallager, (2002). Communication over Fading Channels with Delay Constraints, IEEE Trans. Information Theory, vol. 48, no. 5, pp. 1135-1149.
16. A. Fu, E. Modiano, and J. N. Tsitsiklis, (2006). Transmission scheduling over a fading channel with energy and deadline constraints, IEEE Trans. Wireless Commun., vol. 5, no. 3, pp. 630-641.
17. Murtaza Zafer, Eytan Modiano, (2009): Minimum Energy Transmission Over a Wireless Channel With Deadline and Power Constraints, IEEE Transactions on Automatic Control, vol 54, 12.
18. P.Mohana Shankar. Fading and Shadowing in Wireless Systems. Springer New York. (2012), ISBN 978-1-4614-0366-1
19. McLachlan, G., and D. Peel.(2000). *Finite Mixture Models.* Hoboken, NJ: John Wiley & Sons, Inc.
20. Selim, B., Alhussein, O., Muhaidat, S., Karagiannidis, G.K., and Liang, J,(2016). Modeling and analysis of wireless channels via the mixture of Gaussian distribution, IEEE Trans. Veh. Technol., 2016, vol 65, (10), pp. 8309-8321.
21. Dijkstra, E. W. (1959). "A note on two problems in connexion with graphs" Numerische Mathematik. 1: 269-271.